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根据两个关于岩石强度的标准，构建莫尔极限的包络圈

CONSTRUCTION OF THE ENVELOPE OF THE MOHR'S LIMIT CIRCLES BASED ON TWO CRITERIA FOR THE STRENGTH OF ROCKS

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注解。 在坐标"正常 - 剪切张力"的莫尔图上的极限圆的包络被视为岩石强度的护照。 为了构造包络，使用了最大和最小主应力之间的关系，以两种不同的强度标准的形式呈现。 其中一个是众所周知的Hoek-Brown标准，另一个是T.B.最近提出的。 Duyshenaliev和K.T.Koichumanov。 当考虑A.N.的实验数据时，验证了这些标准的适用性。 通过在三轴压缩条件下测试各种岩石的圆柱形试样以及在单轴拉伸的情况下获得的Stavrogin。

关键词：岩石强度，极限应力圆包络线，强度准则，主应力，莫尔图。

Annotation. *The envelope of limit circles on the Mohr's diagram in the coordinates "normal - shear tensions" is taken as a passport of rock strength. To constructing the envelope there were used the relationship between the maximum and minimum principal stresses, presented in the form of two different strength criteria. One of them is the widely known Hoek-Brown criterion, the other is proposed relatively recently by T.B. Duyshenaliev and K.T. Koichumanov. The applicability of these criteria is verified when considering the experimental data of A.N. Stavrogin obtained by testing cylindrical specimens of various rocks under conditions of triaxial compression, as well as in the case of uniaxial tension.*

Key words: *rock strength, envelope of limit stress circles, strength criterion, principal stresses, Mohr's diagram.*

The stress state of rocks in the massif is modeled by testing standard cylindrical specimens on installations using the Kárman scheme, when the ratio between the axial compressive stress σ_1 and the main stresses σ_2 and σ_3 from uniform lateral pressure is: : $\sigma_1 > \sigma_2 = \sigma_3$. In this case, the compressive stresses are considered positive; their ratio at the time of destruction is told on the strength of the rocks.

Stresses σ_1 and σ_3 can take many values, and it is impossible to carry out the whole complex of experiments with different ratios of these components. Therefore, various methods of calculation are being developed [1], which can be used to estimate the degree of danger of the stress state by the postulated dependencies between the main stresses, i.e. predict the strength properties of materials at the time of destruction.

According to Mohr the stress conditions at which the material is destroyed can be represented by limiting circles in the coordinates $\sigma - \tau$ (normal - shear stress):

$$\left(\frac{\sigma_1+\sigma_3}{2} - \sigma\right) + \tau^2 = \left(\frac{\sigma_1-\sigma_3}{2}\right)^2 \quad (1)$$

In that case the main stresses σ_1, σ_3 correspond to the moment of onset of destruction.

We preliminarily indicate the following: the Mohr's equation of circles can also be represented in a form different from (1):

$$\varphi(\sigma,\tau,c) = \sigma^2 + \tau^2 - (1+c)\sigma_1\sigma + c\sigma_1^2 = 0, \quad (2)$$

In that case as a parameter of the family of circles there is a parameter $c = \sigma_3 / \sigma_1$ - is a type of stress state.

In this case, the coordinates of the envelope (according to the theorem on its existence) after the corresponding transformations are:

$$\sigma = \frac{\sigma_1(\sigma_1+2c(\sigma_1)_c)}{(\sigma_1+(1+c)(\sigma_1)_c)}, \quad \tau = \frac{(1-c)\sigma_1\sqrt{(\sigma_1+c(\sigma_1)_c)\cdot(\sigma_1)_c}}{\sigma_1+(1+c)\cdot(\sigma_1)_c}, \quad ((\sigma_1)_c = \partial\sigma_1/\partial c) \quad (3)$$

Equation (2) in the principal tensorial axes of the stress is a quadratic algebraic equation, from whose invariants it can be concluded that it is a hyperbola equation. Using this property of equation (2), in the monograph [2], the relationship between stresses σ_1 and σ_3 is presented in the form:

$$\sigma_3 = A + \sqrt{\sigma_1^2 + B^2} \quad (4)$$

In which A and B are material constants to be determined for specific materials.

Thus, formula (4) is one of the possible criteria for the strength of rocks.

The definition of the constants A and B can be done by using the parameters and the formula (4), the stress σ_1 is represented as:

$$\sigma_1(c) = \frac{-Ac+\sqrt{A^2-(1-c^2)B^2}}{1-c^2} \tag{5}$$

In the formula (5), the stress σ_1 can be selected for any two types of stress state, carried out in the experiment. Then these stresses will be "reference points" for determining the constants A and B. One of them is advisable to take at c=0 (σ_1 (0)= σ_1 c), the second is at c_{max} from the available experimental data.

The dependence (2) (with the constants A and B) makes it possible to determine the ultimate tensile strength:

$$\sigma_p = A + B \tag{6}$$

The criterion similar to (2) in structure for undisturbed rocks was proposed in [3] in the form:

$$\sigma_1 = \sigma_3 + \sqrt{A_h\sigma_3 + B_h{}^2} \tag{7}$$

Relation (7) is a quadratic algebraic equation:

$$\sigma_1^2 - 2\sigma_1\sigma_3 + \sigma_3{}^2 - A_h\sigma_3 - B_h{}^2 = 0 \tag{8}$$

In accordance with the classification of the quadratic equation [4], the equation (8) (judging by its invariants) is a parabola equation.

As in the consideration of criterion (2), using the uniaxial compression strength and stress for a specific type of stress state (c_{max}) as "reference points", we obtain the expressions for the constants A_h and B_h6:

$$B_h = \sigma_c\,;\ \ A_h = \frac{\sigma_1^2(1-c)^2-\sigma_c{}^2}{\sigma_1 c} \tag{9}$$

With the known constants A_h and B_h, for construction in the case of a triaxial compression of Mohr's circles, an envelope to them and a tensile strength, we obtain:

$$\sigma_1 = \frac{A_h c+\sqrt{A_h{}^2c^2+4(1-c)^2B_h{}^2}}{2(1-c)^2} \tag{10}$$

$$\sigma_p = \frac{A_h-\sqrt{A_h{}^2+4\sigma_c{}^2}}{2} \tag{11}$$

To test the strength criteria (2) and (10), experimental data from triaxial compression tests presented in the monograph [4] were used. As an example in fig. 1 shows the corresponding comparisons for talcum peach

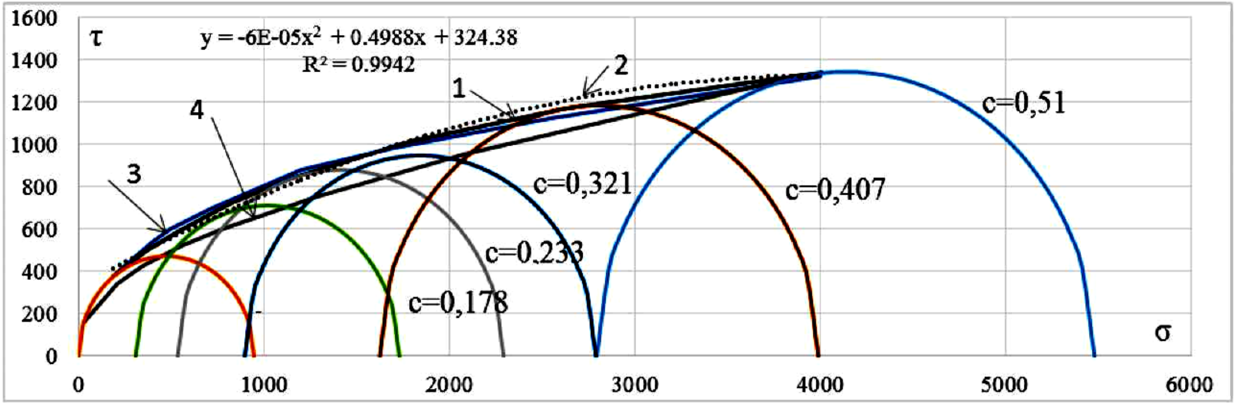

Fig.1. *Mohr's circles and envelopes for talcum peach*
1 - Calculated envelope when using the strength criterion (2);
2-approximation by the trend line envelope 1;
3- empirical envelope [4];
4- calculated envelope when using the strength criterion (10)

Mohr's circles are constructed from experimental data; these circles (see figure 1) almost coincide with those calculated using the strength criterion (2) in almost all types of stress state (c), except for a slight deviation of c = 0.178, c = 0.321.

As can be seen from fig. 1, the strength of a given rock is better reflected by criterion (2) than criterion (7). In addition to this rock, sandstone D-8, Sandstone P-03, Marble II, Limestone (Estonian), Diabase were also investigated. This situation is observed for all the above listed rocks.

According to criterion (2) and formula (6) for talcum peach is obtained: $A = -3815.35$, $B = 3696.5, \sigma_p = -118.88.$ The experimental value of tensile strength $\sigma_p = -130$ (hereinafter, the dimension of stresses and material constants is presented in MPa*9, 81^{-1}).

According to the criterion (7), using the formula (9), for the breed under consideration, $A_h = 2260,37$; $B_h = 945$; $\sigma_p = -343,02$. This value for σ_p differs significantly from its experimental value.

The experimental data of T. Karman for Carrara marble [5] are also considered. An envelope has been constructed for stress limit circles according to the two strength criteria presented above, which is shown in Fig. 2

This study shows that the Duyshenaliev – Koichumanov strength criterion reflects the strength properties of the considered rocks better than the Hoek – Brown criterion, both in cases of non-uniform triaxial compression and uniaxial tension. As follows from the equation for the envelope represented by the trend line, this equation (according to its invariants) is a parabola equation.

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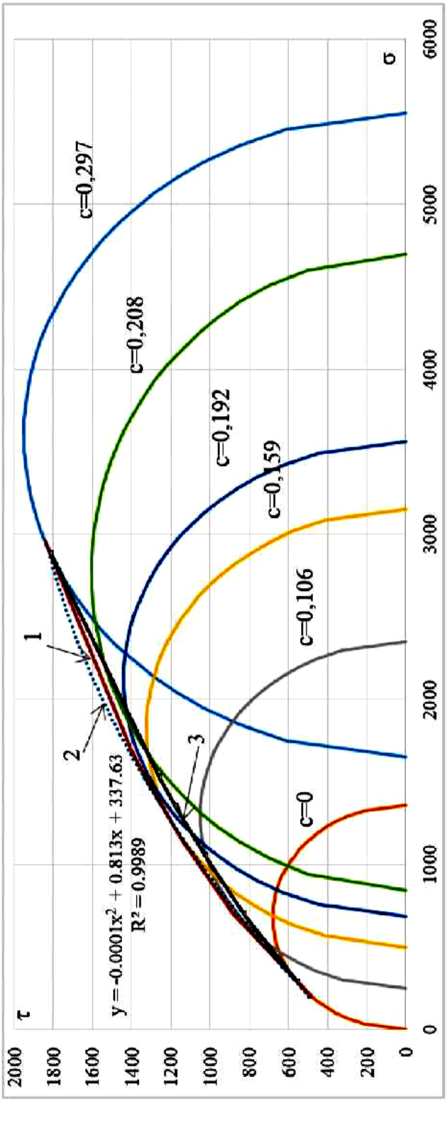

Fig.2. *Mohr's circles and envelopes to them for Carrara marble.*
1 - Calculated envelope when using the strength criterion (2);
2-approximation by the trend line envelope 1;
3- calculated envelope when using the strength criterion (10).